AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 2B

Monday 27 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A stone, of mass $0.3\,\text{kg}$, is thrown with a speed of $8\,\text{m s}^{-1}$ from a point at a height of 5 metres above a horizontal surface.

(a) Calculate the initial kinetic energy of the stone.

[2 marks]

(b) (i) Find the kinetic energy of the stone when it hits the surface.

[3 marks]

(ii) Hence find the speed of the stone when it hits the surface.

[2 marks]

(iii) State one modelling assumption that you have made.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	**Answer space for question 1**

2 A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons.

The unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = (8t - t^4)\mathbf{i} + 6\mathrm{e}^{-3t}\,\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t.

[2 marks]

(b) The mass of the particle is $2\,\mathrm{kg}$.

(i) Find an expression for the force $\mathbf{F}$ acting on the particle at time t.

[2 marks]

(ii) Find the magnitude of $\mathbf{F}$ when $t = 1$.

[3 marks]

(c) Find the value of t when $\mathbf{F}$ acts due south.

[2 marks]

(d) When $t = 0$, the particle is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ metres.

Find an expression for the position vector, $\mathbf{r}$ metres, of the particle at time t.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

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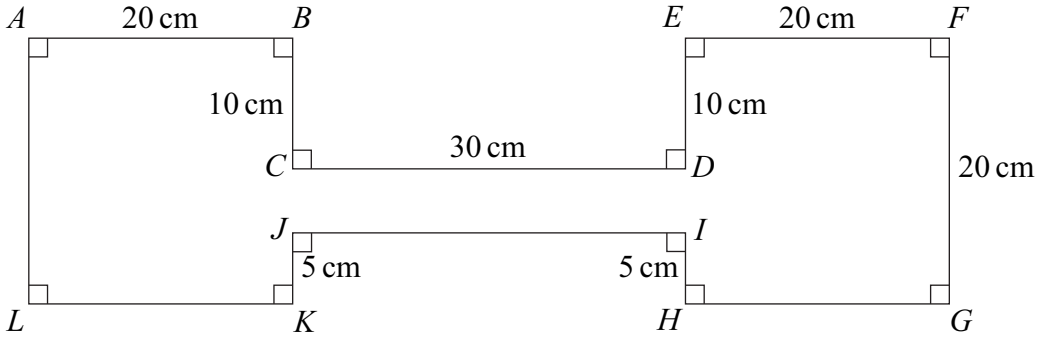
3 The diagram shows a uniform lamina $ABCDEFGHIJKL$.



(a) Explain why the centre of mass of the lamina is $35\,\text{cm}$ from AL.

[1 mark]

(b) Find the distance of the centre of mass from AF.

[4 marks]

(c) The lamina is freely suspended from A.

Find the angle between AB and the vertical when the lamina is in equilibrium.

[3 marks]

(d) Explain, briefly, how you have used the fact that the lamina is uniform.

[1 mark]

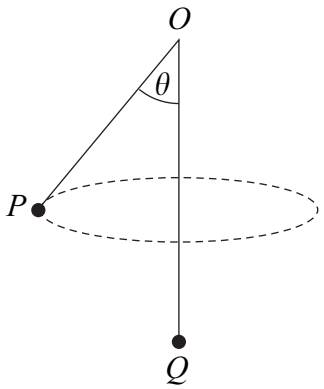
QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE	**Answer space for question 3**

4 A particle P, of mass $6\,\text{kg}$, is attached to one end of a light inextensible string. The string passes through a small smooth ring, fixed at a point O. A second particle Q, of mass $8\,\text{kg}$, is attached to the other end of the string.

The particle Q hangs at rest vertically below the ring, and the particle P moves with speed $5\,\text{m s}^{-1}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



(a) Find the tension in the string.

[1 mark]

(b) Find θ.

[3 marks]

(c) Find the radius of the horizontal circle.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

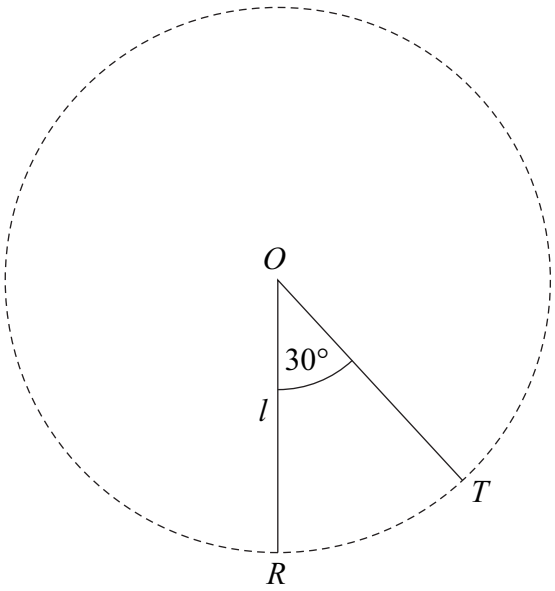
QUESTION PART REFERENCE

Answer space for question 4

5 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l. The particle hangs in equilibrium at the point R vertically below O.

The particle is set into motion with a horizontal velocity u so that it moves in a complete vertical circle with centre O.

The point T on the circle is such that angle ROT is $30°$, as shown in the diagram.



(a) Find, in terms of g, l and u, the speed of the particle at the point T.

[3 marks]

(b) Find, in terms of g, l, m and u, the tension in the string when the particle is at the point T.

[3 marks]

(c) Find, in terms of g, l, m and u, the tension in the string when the particle returns to the point R.

[2 marks]

(d) The particle makes complete revolutions.

Find, in terms of g and l, the minimum value of u.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 A stone, of mass m, falls vertically downwards under gravity through still water. At time t, the stone has speed v and it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - \lambda v$$

[2 marks]

(b) The initial speed of the stone is u.

Find an expression for v at time t.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

7 A uniform ladder, of weight W, rests with its top against a rough vertical wall and its base on rough horizontal ground.

The coefficient of friction between the wall and the ladder is μ and the coefficient of friction between the ground and the ladder is 2μ.

When the ladder is on the point of slipping, the ladder is inclined at an angle of θ to the horizontal.

(a) Draw a diagram to show the forces acting on the ladder.

[2 marks]

(b) Find $\tan\theta$ in terms of μ.

[7 marks]

QUESTION PART REFERENCE

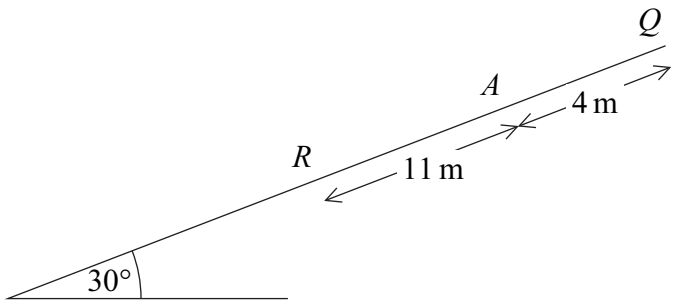
Answer space for question 7

QUESTION PART REFERENCE	**Answer space for question 7**

8 A particle, P, of mass $5\,\text{kg}$ is placed at the point A on a rough plane which is inclined at $30°$ to the horizontal.

The points Q and R are also on the surface of the inclined plane, with $QR = 15$ metres. The point A is between Q and R so that $AQ = 4$ metres and $AR = 11$ metres.

The three points Q, A and R are on a line of greatest slope of the plane.



The particle is attached to two light elastic strings, PQ and PR.

One of the strings, PQ, has natural length 4 metres and modulus of elasticity $160\,\text{N}$, the other string, PR, has natural length 6 metres and modulus of elasticity $120\,\text{N}$.

The particle is released from rest at the point A.

The coefficient of friction between the particle and the plane is 0.4 .

Find the distance of the particle from Q when it is next at rest.

[8 marks]

QUESTION PART REFERENCE

Answer space for question 8

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END OF QUESTIONS

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